Dephasing and weak localization in disordered Luttinger liquid

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We study the transport properties of interacting electrons in a disordered quantum wire within the framework of the Luttinger liquid model. The conductivity at finite temperature is nonzero only because of inelastic electron-electron scattering. We demonstrate that the notion of weak localization is applicable to the strongly correlated one-dimensional electron system. We calculate the relevant dephasing rate, which for spinless electrons is governed by the interplay of electron-electron interaction and disorder, thus vanishing in the clean limit.

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Mesoscopics of strongly correlated electron systems has emerged as an area of great interest to both experimental and theoretical communities working in the field of nanoscale physics. Recently, progress in manufacturing of nanodevices has paved the way for systematic transport measurements on narrow quantum wires with a few or single conducting channels. Most prominent examples of these are semiconductor cleaved-edge quantum wires [1], carbon nanotubes [2], and quantum Hall edges running in opposite directions and interconnected by means of tunneling [3, 4]. On the theoretical side, the challenge is to expand the ideas that have been developed for mesoscopic disordered systems on one side and for strongly correlated clean systems on the other.

Much attention has been focused on the interplay between the interaction effects and disorder-induced localization in diffusive systems of low dimensionality D [5]. A key concept in the localization theory of a disordered Fermi liquid is that of the dephasing rate τ_ϕ^{-1} due to electron-electron (e-e) inelastic scattering. It has been established that a weak-localization (WL) correction to the Drude conductivity of a diffusive system behaves as $\tau_\phi^{(2-D)/2}$ ($\ln \tau_\phi$ for $D = 2$) and thus diverges with lowering T for $D \leq 2$, leading to strong Anderson localization.

This paper is concerned with transport in one dimension (1D), where e-e correlations drive a clean system into the non-Fermi liquid state known as Luttinger liquid (LL) [6]. One more peculiarity of the single-channel 1D system is that the ballistic motion on short scales crosses over in the absence of interaction directly to the localization regime, with no diffusive dynamics on intermediate scales. The main question we address is how the conductivity $\sigma(T)$ behaves in a disordered LL. It appears that a key piece of transport theory as regards the WL and the interaction-induced dephasing in a strongly correlated 1D system is missing. Most authors to date (e.g., [7, 8]) have suggested that the dephasing length that controls localization effects in a disordered LL is $L_T = u/T$ (throughout the paper $\hbar = 1$), where u is the plasmon velocity. According to this approach, the interference effects get strong with lowering T at $L_T \sim \xi$, where ξ is the localization length. An alternative approach [9, 10] is predicated on the assumption that the dephasing rate is determined by the single-particle properties of a clean LL. On top of that, one might think that since in the case of linear dispersion the interacting electron system can be equivalently represented in terms of *noninteracting* bosons, the interaction should not induce any dephasing at all. The conductivity would then be exactly zero at any T. As we argue below, none of the approaches captures the essential physics of dephasing in the conductivity of a disordered 1D system.

We begin by considering the Drude conductivity under the condition that τ_ϕ is much shorter than the transport time of elastic scattering off disorder τ and the Anderson-localization effects are completely destroyed. For simplicity, we assume that interaction is weak and short-ranged. We also assume that $\epsilon_F \tau \gg 1$, where ϵ_F is the Fermi energy. To leading order in $\tau_\phi/\tau \ll 1$, the conductivity is given by the Drude formula $\sigma^{\rm D} = e^2 \rho v_F^2 \tau$ ($\rho = \partial n/\partial \mu \simeq 1/\pi v_F$ is the compressibility, v_F the Fermi velocity) and depends on T through a T-dependent renormalization of the static disorder [7, 11]:

$$\tau_0/\tau = (\Lambda/T)^{2\alpha'} , \qquad (1)$$

where $\alpha' = \eta_s^{-1}[1 - (1 + 2\eta_s\alpha)^{-1/2}] \simeq \alpha = V_f/2\pi v_F > 0$ characterizes the strength of repulsive interaction between electrons (we assume that $\alpha \ll 1$), $\eta_s = 1$ or 2 for spinless or spinful electrons, respectively; V_f is the Fourier transform of a forward-scattering potential, τ_0^{-1} the scattering rate at $\alpha = 0$. For $\alpha \ll 1$, the ultraviolet cutoff Λ may be put equal to ϵ_F. The exponent in Eq. (1) is given by the bare interaction constant (the one in a clean system) since the running coupling constant [7] is not renormalized by disorder for $T\tau \gg 1$. The renormalization (1) is similar to the Altshuler-Aronov corrections [5] in higher dimensionalities. At this level, the only peculiarity of LL as compared to higher D is that the renormalization of τ is more singular and necessitates going beyond the Hartree-Fock (HF) approach [12].

The renormalization of τ stops with decreasing T at $T\tau \sim 1$. This condition gives the zero-T localization

length $\xi \propto \tau_0^{1/(1+2\alpha')}$, but does not correctly predict the onset of localization (determined by the condition $\tau/\tau_\phi \sim 1$), in contrast to the argument made in Refs. 7, 8. This can be seen, in particular, by noting that the temperature $T \sim \tau^{-1}$ does not depend on the strength of interaction for small α, whereas it is evident that for noninteracting electrons $\sigma(T) = 0$ for any T. The error appears to be based on the renormalization-group equations [7], which treat scalings with length and u/T as interchangeable. While this approach is justified for the "elastic renormalization" of τ, Eq. (1), it does not properly account for the WL and misses all effects associated with dephasing by construction.

Let us now turn to the calculation of τ_ϕ^{-1}. Our approach is closely related to that for higher dimensionalities [5] and it is instructive to first analyze the Golden Rule expression for the e-e collision rate following from the Boltzmann kinetic equation:

$$\frac{1}{\tau_{\rm ee}(\epsilon)} = \int d\omega \int d\epsilon' K_\omega (f^h_{\epsilon-\omega} f_{\epsilon'} f^h_{\epsilon'+\omega} + f_{\epsilon-\omega} f^h_{\epsilon'} f_{\epsilon'+\omega}), \quad (2)$$

where K_ω is the kernel of the e-e collision integral, f_ϵ is the Fermi distribution function, and $f^h_\epsilon = 1 - f_\epsilon$. Peculiar to 1D are highly singular contributions to K_ω related to scattering of electrons moving in the same direction. Indeed, consider a perturbative expansion of K_ω to second order in α in a clean LL. For simplicity, let α be a momentum-independent constant. At the Fermi level ($\epsilon = 0$), $1/2\tau_{\rm ee} = \eta_s(\Sigma^H_{++} + \Sigma^H_{+-}) + \Sigma^F$, where

$$\Sigma^H_{+\pm} \simeq \pi\alpha^2 v_F T \int_{|\omega| \lesssim T} d\omega \int dq\, \delta(\omega - v_F q)\delta(\omega \mp v_F q) \quad (3)$$

are the Hartree terms for scattering of two electrons from the same $(++)$ or different $(+-)$ chiral spectral branches and $\Sigma^F = -\Sigma^H_{++}$ is the exchange term. One sees that the contribution of Σ^H_{++} is diverging. For spin-polarized electrons it is, however, canceled by the exchange interaction. The remaining term Σ^H_{+-} is determined by $\omega, q \to 0$ and is given by $2\Sigma^H_{+-} = \pi\alpha^2 T$. Already the perturbative expansion demonstrates a qualitative difference between two cases of spinless and spinful electrons.

Below we concentrate on the spinless case. Terms of higher order in α may then be neglected due to the order-by-order cancellation of the singular Hartree and exchange contributions, so that we obtain

$$\tau_{\rm ee}^{-1} = \pi\alpha^2 T\ . \quad (4)$$

It is instructive to compare this collision rate in a clean LL with the damping of the retarded single-particle Green's function in the (x,t) representation, $g^R(x,t) = 2i\theta(t){\rm Im}\,\bar{g}(x,t)$, where (for right-movers)

$$\bar{g} = \frac{T/2u}{\sinh[\pi T(\frac{x}{u} - t + i0)]} \frac{(\pi T/\Lambda)^{\alpha_b}}{[\sinh(\pi T\tau_-)\sinh(\pi T\tau_+)]^{\alpha_b/2}}\ ,$$

$\tau_\pm = \pm(t - i/\Lambda) + x/u$, and $\alpha_b = [(1+2\alpha)^{1/4} - (1+2\alpha)^{-1/4}]^2/2 \simeq \alpha^2/2$ for $\alpha \ll 1$. The temporal decay $\exp(-\pi\alpha_b T t)$ of the residue

$$(x - ut)g^R(x,t)|_{x\to ut} \propto \sinh^{-\alpha_b/2}(2\pi T t) \quad (5)$$

for $t \to \infty$ agrees with Eq. (4) to order α^2: $\pi\alpha_b T = 1/2\tau_{\rm ee}$. The e-e scattering thus manifests itself in that it cuts off the power-law decay in Eq. (5), characteristic of the zero-T limit.

The notion of dephasing associated with the behavior of the single-particle Green's function (5) makes sense in a clean LL in the ring geometry, where this kind of dephasing governs the decay rate $(\tau_\phi^{\rm AB})^{-1}$ of Aharonov-Bohm (AB) oscillations [10, 13]. However, as far as $\sigma(T)$ is concerned, the significance of the dephasing rate in the limit of high T is that it cuts off a WL correction $\sigma^{\rm wl}$ to the Drude conductivity [14]. At this point, it is important to note that the characteristic energy transfer in Eq. (2), $\omega_0 \sim \tau^{-1}$, is much smaller than $\tau_{\rm ee}^{-1}$ in the WL regime. It suggests that the dephasing rate $1/\tau_\phi^{\rm wl}$ that determines $\sigma^{\rm wl}$ requires a self-consistent cutoff in Eq. (2) at $\omega \sim 1/\tau_\phi^{\rm wl}$ (since soft inelastic scattering with $qv_F, \omega \ll 1/\tau_\phi^{\rm wl}$ does not affect $\sigma^{\rm wl}$ [15]), and so is parametrically different from the one in Eq. (4), $\tau_\phi^{\rm wl} \neq \tau_{\rm ee}$.

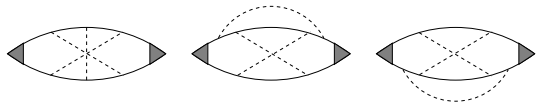

FIG. 1: Diagrams describing the leading WL correction to the conductivity of Luttinger liquid for $\tau_\phi^{\rm wl} \ll \tau$. The dashed lines represent backscattering off impurities. The current vertices are dressed by impurity ladders. The diagrams are understood as fully dressed by e-e interactions.

To evaluate $\sigma^{\rm wl}$ quantitatively, we use a path-integral representation: the leading localization correction in the ballistic limit $\tau_\phi^{\rm wl}/\tau \ll 1$ is related to the interference of electrons scattered by three impurities. The corresponding diagrams are given by a "three-impurity Cooperon" (Fig. 1), which describes the propagation of two electron waves along the path connecting three impurities ("minimal loop") in time-reversed directions. In the absence of interaction, quantum interference processes involving a larger number of impurities sum to exactly cancel the Drude conductivity [16], which spells complete localization. For $\tau_\phi^{\rm wl}/\tau \ll 1$, they only yield subleading corrections through a systematic expansion in powers of $\tau_\phi^{\rm wl}/\tau$.

The dephasing-induced action $S(t, t_a)$ acquired by the Cooperon is accumulated on the classical (saddle-point) path, whose geometry for three impurities if fixed by two length scales, the total length of the path $v_F t$ and the distance between two rightmost impurities $v_F t_a \le v_F t/2$ (Fig. 2). The WL correction can then be represented as

$$\sigma^{\rm wl} = -2\sigma^{\rm D}\int_0^\infty dt \int_0^\infty dt_a P(t, t_a) \exp\left[-S(t, t_a)\right]\ , \quad (6)$$

where $P(t,t_a) = (1/8\tau^2)\exp(-t/2\tau)\theta(t-2t_a)$ is the probability density of return to point $x=0$ after two reflections at points $x = v_F t_a$ and $x = -v_F(t/2 - t_a)$. The contribution S_{ij} to the dephasing action associated with inelastic interaction between electrons propagating along the paths $x_i(t)$ and $x_j(t)$ is obtained similarly to higher dimensionalities [5, 15]:

$$S_{ij} = -T\int\frac{d\omega}{2\pi}\int\frac{dq}{2\pi}\int_0^t dt_1\int_0^t dt_2\,\frac{1}{\omega}\mathrm{Im}V_{\mu\nu}(\omega,q) \times\exp\{iq\,[\,x_i(t_1)-x_j(t_2)\,]-i\omega(t_1-t_2)\}\ . \quad (7)$$

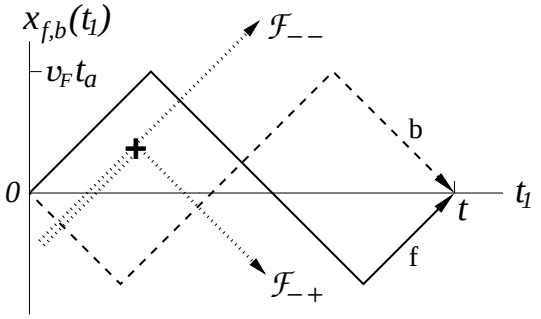

FIG. 2: Illustration of electron dynamics governing the WL and dephasing: Time-reversed paths $x_f(t_1)$ (solid) and $x_b(t_1) = x_f(t-t_1)$ (dashed) on which the interaction-induced action S that yields dephasing of the Cooperon is accumulated. Dotted lines: the propagation of dynamically screened interaction. The interaction may change the direction of propagation upon scattering off disorder (as marked by a cross). Each interaction line gives a contribution to S proportional to $(N_f - N_b)^2$, where $N_{f,b}$ is the number of its intersections with the forward (f) and backward (b) paths. One sees that $N_f \neq N_b$ only due to impurity scattering in the interaction propagator. Interaction and electron lines lying on top of each other do not yield dephasing because of the HF cancellation.

The main steps in the derivation of Eq. (7) are: (i) the random-phase approximation (RPA), (ii) the independent averaging of each of the RPA bubbles over disorder, and (iii) treatment of thermal electromagnetic fluctuations through which electrons interact with each other as a classical field. This approach is justified if the characteristic energy transfer is much smaller than T, which is the case for $T\tau_\phi^{\rm wl} \gg 1$. Because of the HF-cancellation of the bare interaction between electrons from the same chiral branch, the dynamically screened (retarded) interaction $V(\omega,q)$ in Eq. (7) should be calculated as if the bare coupling is only present for electrons moving in opposite directions [17]. As a result, $V_{\mu\nu}$ acquires the chiral indices $\mu = \mathrm{sgn}\,\dot{x}_i$, $\nu = \mathrm{sgn}\,\dot{x}_j$. Expanding $V_{\mu\nu}$ to second order in α we have $\mathrm{Im}V_{\mu\nu} = -\pi\alpha^2 v_F\omega\mathcal{F}_{\mu\nu}$, where $\mathcal{F}_{\mu\nu} = 4\mathrm{Re}\,\mathcal{D}_{-\mu,-\nu}$ and $\mathcal{D}_{\mu\nu}$ are the particle-hole propagators for noninteracting electrons. The action S_{ij} can then be written in a simple form:

$$S_{ij} = \pi\alpha^2 v_F T\int_0^t dt_1\int_0^t dt_2\,\mathcal{F}_{\mu\nu}[\,x_i(t_1)-x_j(t_2),t_1-t_2\,]\ ,$$

where, to first order in τ^{-1}, $\mathcal{F}_{\mu\nu}(x,t)$ read

$$\mathcal{F}_{++}(x,t)\ \simeq\ \delta(x+v_F t)\,(1-|t|/2\tau)\ , \quad (8)$$
$$\mathcal{F}_{+-}(x,t)\ \simeq\ \theta(v_F^2t^2-x^2)/4v_F\tau\ , \quad (9)$$

and $\mathcal{F}_{--}(x,t) = \mathcal{F}_{++}(-x,t)$, $\mathcal{F}_{-+}(x,t) = \mathcal{F}_{+-}(x,t)$. The total action is given by $S = 2(S_{\rm ff} - S_{\rm fb})$, where f and b stand for "forward" and "backward" time-reversed paths (Fig. 2).

Calculating first S for $\tau^{-1} = 0$ we have $S_{\rm ff} = S_{\rm fb} = \pi\alpha^2 Tt/2$. One sees that $S_{\rm ff}$ reproduces the AB dephasing, Eqs. (4),(5). The subtle point, however, is that for $\tau^{-1} = 0$ the self-energy processes ($S_{\rm ff} + S_{\rm bb}$) are exactly canceled in S by the vertex corrections ($S_{\rm fb} + S_{\rm bf}$), i.e., $S = 0$ in a clean LL. Hence, the dephasing in Eq. (6) is only due to the dressing of the dynamically screened interaction by impurities. To order $S \sim \mathcal{O}(\tau^{-1})$ we obtain

$$S(t,t_a) = 2\pi\alpha^2\,T\,t_a\,(t-2t_a)\,/\tau\ . \quad (10)$$

The dephasing vanishes for $t_a = 0, t/2$ since in these cases the Cooperon is not distinguishable from the diffuson.

Substituting Eq. (10) in Eq. (6) we find for $\tau_\phi^{\rm wl}/\tau \ll 1$:

$$\sigma^{\rm wl} = -\frac{1}{4}\,\sigma^{\rm D}\left(\frac{\tau_\phi^{\rm wl}}{\tau}\right)^2\ln\frac{\tau}{\tau_\phi^{\rm wl}} \propto \frac{1}{\alpha^2 T}\,\ln(\alpha^2 T)\ , \quad (11)$$

where

$$\frac{1}{\tau_\phi^{\rm wl}} = \alpha\left(\frac{\pi T}{\tau}\right)^{1/2}\ , \quad T \gg T_1 = \frac{1}{\alpha^2\tau}\ . \quad (12)$$

Note that $1/\tau_\phi^{\rm wl}$ vanishes in the clean limit [18], in contrast to the total e-e scattering rate, Eq. (4). It is worth mentioning that the T dependence of $\sigma(T) = \sigma^{\rm D}(T) + \sigma^{\rm wl}(T)$ is dominated by the WL term rather than by $\sigma^{\rm D}(T)$ for $T \ll T_1/\alpha$. The scale T_1 marks the temperature below which the localization effects become strong. These results are illustrated in Fig. 3.

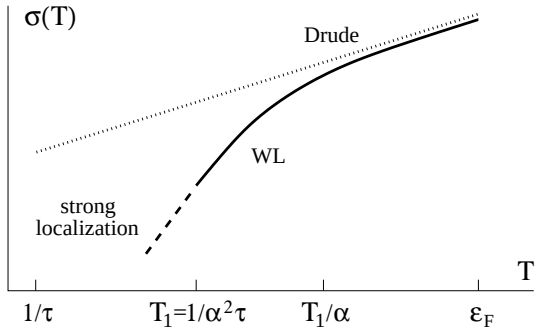

FIG. 3: Schematic behavior of $\sigma(T)$ on the log-log scale. Dotted line: the T-dependent Drude conductivity [7, 11]. Below T_1/α the WL correction, Eq. (11), dominates $d\ln\sigma/d\ln T$. Below T_1 the localization becomes strong [13].

Before closing the paper, let us briefly mention a few extensions [13] of our results.

(i) *"All-in-one" approach.* In effect, Eqs. (11),(12) were derived in two steps: first the static disorder was renormalized by virtual processes with energy transfers in the range between T and Λ [LL renormalization, Eq. (1)] and then the dephasing rate due to real processes with transfers smaller than T (similarly to Fermi-liquid dephasing) was calculated for electrons scattered by the renormalized disorder. Alternatively, the virtual and real transitions can be treated on an equal footing by means of the "functional bosonization" [19]. Including disorder in the bosonic propagators we reproduced Eqs. (11),(12) by this method as well.

(ii) *Spin.* In contrast to the spinless case, for $\alpha \ll 1$ the main contribution to τ_ϕ^{-1} of spinful electrons comes from scattering of electrons from the same chiral branch on each other. In the clean limit, the perturbative expansion of τ_ϕ^{-1} in powers of α is diverging at each order, as in Eq. (3). The most singular terms in τ_ϕ^{-1} can be summed by means of the RPA and written after the HF cancellation in the form of $2\Sigma^H_{++}$, Eq. (3), with v_F in one of the δ-functions being replaced by the plasmon velocity u. Due to the HF cancellation, the latter is taken here as if electrons were spinless, i.e., from $(u/v_F)^2 = 1 + 2\eta_s\alpha$ with $\eta_s = 1$. For $\alpha \ll 1$ this gives

$$\frac{1}{\tau_\phi} = 2\pi\alpha^2 \frac{v_F}{|u - v_F|} T \simeq 2\pi\alpha T \ , \quad T \gg T_1^s = \frac{1}{\alpha\tau} \ . \quad (13)$$

This result agrees with the behavior of $\ln|(x - ut)^{1/2} g^R(x,t)| \to -t/2\tau_\phi$ at $x = ut$ for spinful electrons in a clean LL, similarly to Eq. (5). In contrast to spinless electrons, Eq. (13) describes the dephasing in both the AB and WL setups; in the latter case, we have $\sigma^{\rm wl} \sim -\sigma^{\rm D}\tau_\phi/\tau$. Below T_1^s the localization sets in.

(iii) *Low temperature.* In this paper, we have investigated transport at sufficiently high $T \gg T_1$, when $\tau_\phi^{\rm wl}/\tau \ll 1$. Below T_1 the effects of Anderson localization become strong. With lowering T they lead first to an intermediate regime of "power-law hopping" [20], where $\sigma(T) \sim \sigma^{\rm D}\tau/\tau_\phi$ is a power-law function of T. For still lower T, the system enters the "Anderson-Fock glass" phase, where $\sigma(T)$ vanishes due to the Anderson localization transition in many-body Fock space [13, 21, 22].

In conclusion, we have studied the dephasing of WL in a disordered LL. For spinless electrons, our main result is the WL correction (11), governed by the dephasing rate (12). The latter is parametrically different from the AB dephasing rate, Eq. (4). Our approach provides a framework for systematically studying the mesoscopic phenomena in strongly correlated electron systems.

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